NON-NEWTONIAN BEHAVIOR OF BLOOD IN OSCILLATORY FLOW

ALBERT L. KUNZ *and* NORMAN A. COULTER, JR.

From the Department of Physiology, The Ohio State University, Columbus, and the Section on Biomathematics and Bioengineering, Division of Thoracic Surgery, Department of Surgery, School of Medicine, The University of North Carolina, Chapel Hill

ABSTRACT Sinusoidal oscillatory flow of blood and of aqueous glycerol solutions was produced in rigid cylindrical tubes. For aqueous glycerol, the amplitude of the measured pressure gradient wave form conformed closely to that predicted by Womersley's theory of oscillatory flow, up to Reynolds numbers approaching 2000. Blood differed significantly from aqueous glycerol solutions of comparable viscosity, especially at low frequencies and high hematocrits. As frequency increased, the hydraulic impedance of blood decreased to a minimum at a frequency of about 1–2 CPS, increasing monotonically at higher frequencies. The dynamic apparent viscosity of blood, calculated from Womersley's theory, decreased with increasing flow amplitude. The reactive component of the hydraulic impedance increased with frequency as predicted by theory; the resistive component decreased with increasing frequency, differing from the resistance of a Newtonian fluid which increased with frequency.

INTRODUCTION

The non-Newtonian characteristics of blood have been extensively investigated under steady flow conditions, and the variation of the blood's apparent viscosity with shear rate, hematocrit, and vessel caliber is well-known, if imperfectly understood. Very little is known, however, about the rheology of blood under oscillatory conditions. A theoretical study was made by Taylor (14) but experimental data are meager. In view of the growing interest in pulsatile hemodynamics, a better understanding of pulsatile hemorheology is of basic importance.

The purpose of the study here reported was to compare the behavior of blood of different hematocrits with that of a Newtonian fluid, aqueous glycerol, in oscillatory flow. In particular, information to answer three basic questions was sought:

1. What is the hydraulic impedance of blood compared to that of a Newtonian fluid?
2. What is the apparent viscosity of blood in oscillatory flow?
3. What are the hydraulic resistance and reactance of blood compared to those of a Newtonian fluid?

Indispensible to the experimental answers to these questions has been the powerful theory of oscillatory flow in rigid cylindrical tubes developed by Crandall (3), Lambossy (9), and Womersley (16). This theory provides convenient yardsticks by which to compare quantitatively blood and Newtonian fluids of comparable viscosity.

Since this theory is most extensively developed and best known in the form given it by Womersley, we shall follow his treatment. An excellent presentation is given by McDonald in chapter V (11) and by Fry and Greenfield (5). All symbols are defined in Table I.

METHODS

Sinusoidal flow was produced by a pump, whose displacement was continuously measured and recorded by means of a position transducer (linear variable differential transformer,

TABLE I

LIST OF SYMBOLS

P	= pressure gradient (pressure difference per unit length)
P_m	= amplitude of pressure gradient sinusoid
Q	= instantaneous volume flow
Q_m	= amplitude of flow wave form
r	= radius of tube
d	= diameter
μ	= viscosity of fluid
μ'	= apparent viscosity
f	= frequency (CPS)
ω	= angular frequence (radians/sec)
ϵ'_{10}	= phase angle in Womersley's theory
M'_{10}	= modulus in Womersley's theory
α	= dimensionless frequency parameter = $r\sqrt{\omega\rho/\mu}$
ρ	= density of fluid
$\bar{v}$	= average velocity
Re	= Reynolds number

Schaevitz Engineering, Waltham, Mass.). The (peak-to-peak) stroke volume V being known, the volume flow rate Q could be determined at any instant for a sinusoidal stroke from the relation

$$Q = \pi f V \cos 2\pi f t \tag{1}$$

where f = frequency.

In these experiments, V was constant at 0.204 cc.

Pressures were measured continuously by means of two Statham P23Db pressure transducers (Statham Instruments, Inc., Los Angeles, California) which were coupled to the rigid flow tube via side arms, as indicated in Fig. 1. These were subtracted electronically and recorded. In some experiments a differential electromanometer (Pappenheimer, 12) was used. The flow tube primarily used had a radius of 0.17 cm and an over-all length of 100 cm. Pressure taps were 40 cm apart. Each tap was 30 cm from the nearer end of the tube, to minimize the influence of inlet conditions.

The distal end of the flow tube was coupled to a pressure reservoir, as indicated in the figure. In pilot experiments it was found necessary to apply a pressure to the entire system; otherwise, during the withdrawal phase of the pump cycle, blood gases were drawn out of solution to form bubbles. A pressure of 200 mm Hg applied to the pressure bottle was adequate to prevent this. Since *the pressure difference* was measured and the tubes were rigid, the hydrodynamic situation was not essentially altered. Pilot experiments also showed that the compliance of the system was of critical importance, since compliance "in parallel" with the flow tube shared the flow delivered by the pump. At higher frequencies, under these conditions, a large part of the flow delivered by the pump would go into and out of the compliance. Accordingly, compliance was routinely checked before and after each experiment. Errors due to compliance were estimated to be less than 2% (corresponding to a maximum compliance of 0.0015 cm^3/100 mm Hg).

The density of the fluids used in these experiments was measured by weighing a known quantity of fluid on an analytical balance. The viscosity of the aqueous glycerol solutions was ascertained from a table (Handbook of Chemistry and Physics), using the measured density as argument.

Heparin was used as the anticoagulant for blood. Cells and plasma were separated by centrifugation, and then the cells were resuspended in various volumes of plasma to produce blood samples of different viscosity for study. The hematocrit was determined by centrifugation in Wintrobe tubes for 20 min. at 3800 RPM. The average of two samples was used for each determination.

RESULTS

Womersley's theory for a pressure gradient $P_m \cos \omega t$ predicts a flow rate Q given by

$$Q = P_m \frac{\pi r^4}{\mu} \frac{M'_{10}}{\alpha^2} \sin (\omega t - \epsilon'_{10}). \tag{2}$$

(All symbols are defined in Table I.)

The amplitude Q_m of the flow wave form is then

$$Q_m = P_m \frac{\pi r^4}{\mu} \frac{M'_{10}}{\alpha^2} \tag{3}$$

or

$$P_m = \frac{\mu}{\pi r^4} \frac{\alpha^2}{M'_{10}} Q_m . \tag{4}$$

Similarly, from equation (1), the amplitude Q_m of the flow wave form is

$$Q_m = \pi f V. \tag{5}$$

For a Newtonian fluid of known density and viscosity, oscillating sinusoidally in a tube of known radius with flow amplitude Q_m, all of the quantities on the right-hand side of equation (4) may be determined. (The quantity α^2 may be calculated from the formula $\alpha^2 = r^2\omega p/\mu$; M'_{10} may be determined from Womersley's

Table I, using α as argument.) Hence, theoretical values of P_m may be calculated and compared with experimental values. This is done in Fig. 2 for distilled water and for 60% glycerol. Experimental points lie along the theoretical curves, within the limits of experimental error. Intermediate concentrations of aqueous glycerol showed a similar degree of correlation. It should be noted from equation (5) that in these experiments, as frequency is increased so is flow amplitude.

The maximum Reynolds number[1] for these experiments ranged from 312 for 60% glycerol to 2040 for distilled water. By hydrodynamic criteria, there was no indication of turbulence. The maximum Reynolds number achieved for each solu-

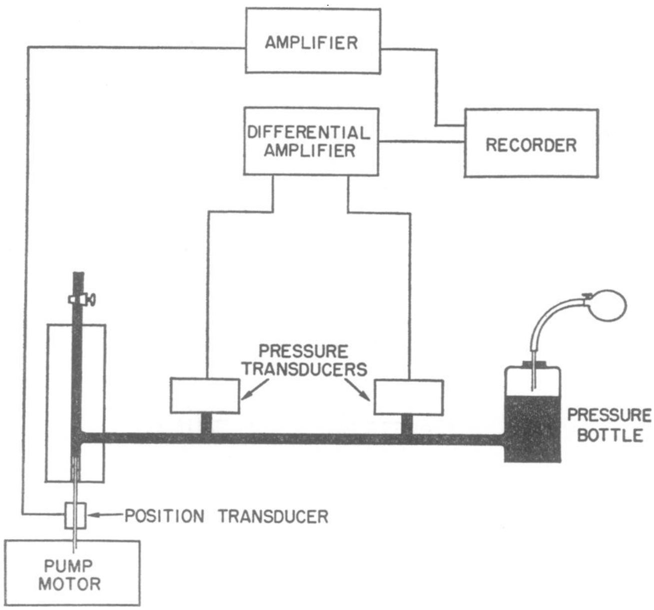

FIGURE 1 Diagram of experimental system. Length of flow tube, 100 cm; distance between pressure transducers, 40 cm; radius of tube, 0.017 cm.

tion is given in Table II. The second highest value for distilled water was 1669; too much significance, therefore, should not be attached to the single value exceeding 2000.

The dynamic pressure flow relation of blood is superficially similar to that of Newtonian fluids when plots of Q_m vs. P_m are made. However, we were unable to obtain quantitative agreement between the experimental curves for blood and the theoretical curves calculated from Womersley's theory for Newtonian fluids of the same density as blood. We tried to do this by introducing different values for viscosity into Womersley's equations. For relatively low values of viscosity, a curve

[1] $Re = \rho vd/\mu$.

could be obtained which approximated the experimental points at low values of Q_m, but which was much too low at higher values of Q_m. For relatively high values of viscosity, a curve could be obtained which approximated the experimental points at high values of Q_m, but which was too high at low values of Q_m.

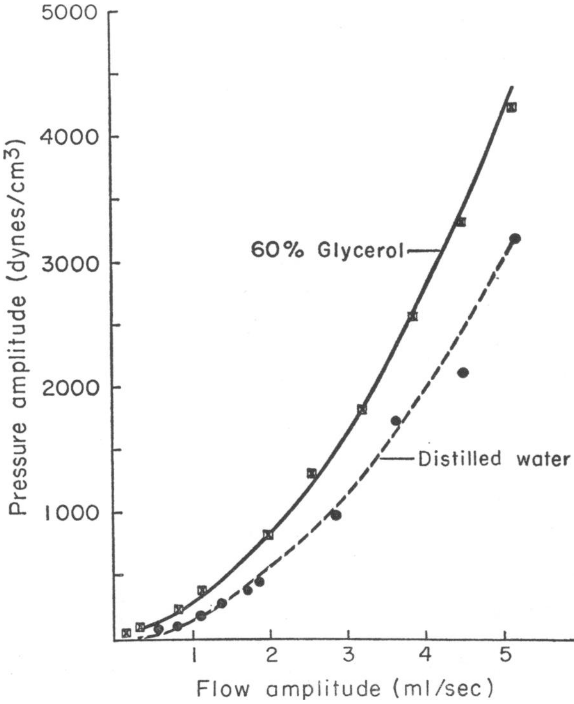

FIGURE 2 Pressure gradient amplitude vs. flow amplitude, 60% aqueous glycerol and distilled water. Smooth curves calculated from Womersley's theory as described in text.

TABLE II

MAXIMUM REYNOLDS NUMBERS OF AQUEOUS GLYCEROL SOLUTIONS IN OSCILLATORY FLOW

Viscosity	Frequency	Maximum Reynolds number
centipoise	*cps*	
0.824	5.7	1669
0.824	7.0	2040
1.356	8.5	1574
2.313	8.8	986
3.131	8.0	676
4.726	8.5	490
7.066	8.0	312

Fig. 3 illustrates the best approximation we were able to make, achieved by selecting an intermediate value of viscosity ($\mu = 0.02862$ poise). The smooth curve, obtained from Womersley's theory for a Newtonian fluid of this viscosity and the same density (1.0985) as blood, is compared to ox blood of 47% hematocrit.

Fig. 4 demonstrates the non-Newtonian behavior of blood in a different way. Here, the modulus of hydraulic impedance ($Z_m = P_m/Q_m$) is plotted against frequency for ox blood of different hematocrits. The curve for 4% hematocrit is similar to theoretical curves for Newtonian fluids and can be used as a basis for

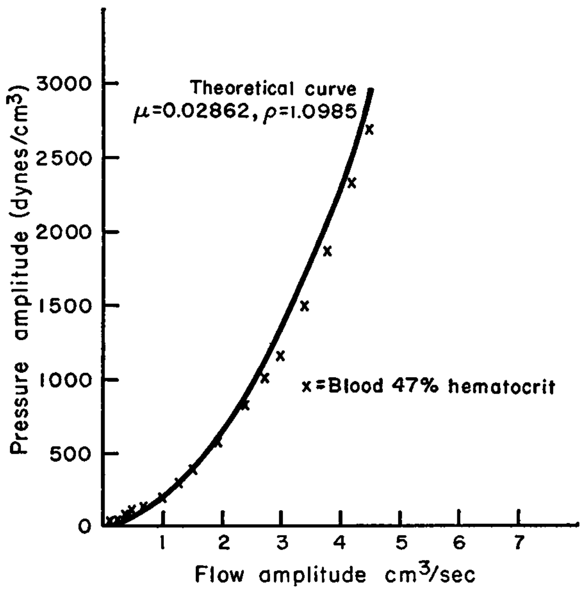

FIGURE 3 Pressure gradient amplitude vs. flow amplitude, ox blood of 47% hematocrit. Theoretical curve calculated from Womersley's theory for a Newtonian fluid of viscosity 0.02862 poise, density 1.0985 gm/cm³.

comparison. It can be seen that blood exhibits an anomalously high hydraulic impedance at low frequencies. The effect is most marked at higher hematocrits.

In studies of blood rheology under steady flow conditions, the apparent viscosity of blood is frequently calculated from Poiseuille's law by defining the apparent viscosity as the viscosity of an equivalent Newtonian fluid which would yield the same volume flow for the same applied pressure difference. Womersley's theory enables us to determine in an analogous way the apparent viscosity of blood under oscillatory conditions. Solving equation (3) for M'_{10}, we have

$$M'_{10} = \frac{\mu\alpha^2}{\pi r^4} \frac{Q_m}{P_m}. \tag{6}$$

Substituting for α^2 its equivalent $r^2\omega\rho/\mu$ and simplifying, we obtain

$$M'_{10} = \frac{2f\rho}{r^2}\frac{Q_m}{P_m}. \tag{7}$$

All the quantities on the right-hand side of equation (5) are measured experimentally. We can, therefore, calculate M'_{10} for each experimental point. Knowing M'_{10}, we can determine α from Womersley's Table I (16).

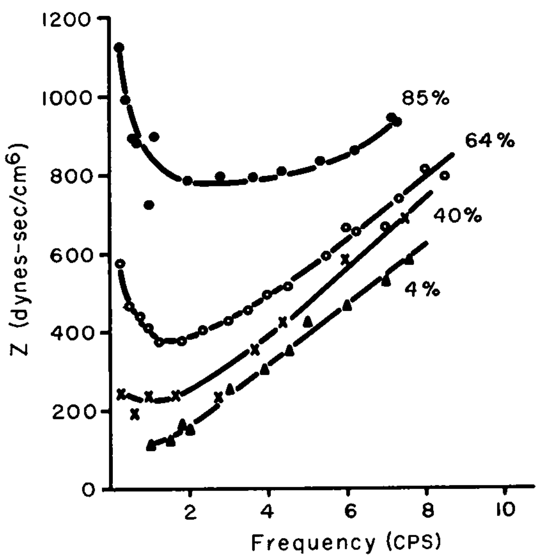

FIGURE 4 Hydraulic impedance vs. frequency, ox blood of different hematocrits. Ordinate: modulus of impedance (dyne-sec/cm⁶); abscissa: frequency (CPS).

The apparent viscosity μ' can then be calculated from the expression

$$\mu' = \frac{r^2\omega_\rho}{\alpha^2}. \tag{8}$$

As in the case of steady flow, the apparent viscosity of blood under oscillatory conditions is the viscosity of an equivalent homogeneous fluid which would yield the same amplitudes of the pressure and flow wave forms. In Fig. 5, the apparent viscosity so calculated is plotted against the maximum flow for blood of different hematocrits. The apparent viscosity decreases with increasing maximum flow in a manner resembling the apparent viscosity flow curves under steady-state conditions (Kumin, 8; Bayliss, 1; Haynes and Burton, 7; and Coulter and Pappenheimer, 2).

For a Newtonian fluid, of course, the viscosity versus frequency characteristic is a straight line parallel to the X axis.

In a rigid tube the hydraulic impedance Z per unit length is given by

$$Z = R + jX \tag{9}$$

where R = hydraulic resistance and X = hydraulic reactance.

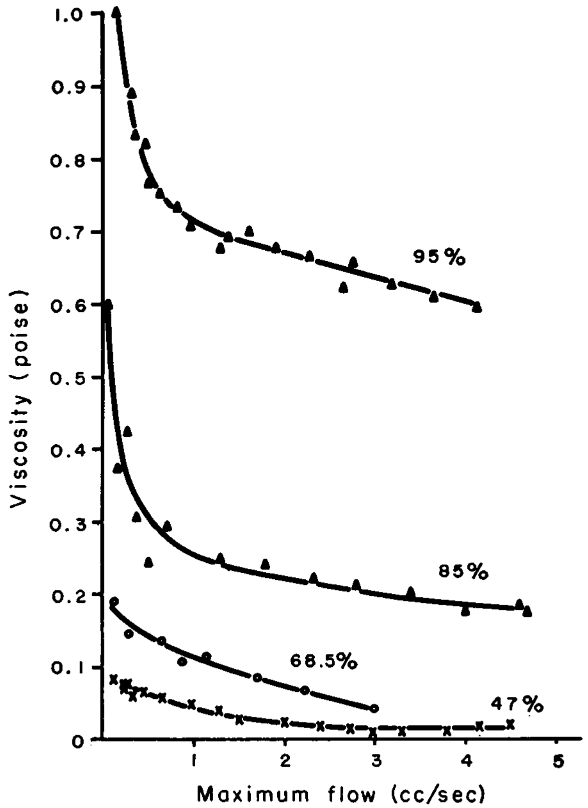

FIGURE 5 Dynamic apparent viscosity of blood of different hematocrits, plotted against flow amplitude. Smooth curves have no theoretical significance.

From Womersley's theory, the hydraulic resistance is

$$R = \frac{\mu \alpha^2 \sin \epsilon_{10}'}{\pi r^2 M_{10}'}. \tag{10}$$

The hydraulic reactance is

$$X = \frac{\mu \alpha^2 \omega \cos \epsilon_{10}'}{\pi r^4 M_{10}'}. \tag{11}$$

In Fig. 6, the hydraulic resistance and reactance of blood of 47% hematocrit is plotted as a function of frequency. For comparison, smooth curves are also drawn to show the hydraulic resistance and reactance of a Newtonian fluid of comparable viscosity ($\mu = 0.04$ poise). It will be noted that the hydraulic reactance of blood differs little from that of a Newtonian fluid, at least under these conditions. The hydraulic resistance, on the other hand, differs significantly. It actually decreases with increasing frequency, eventually passing through a minimum at higher frequencies.

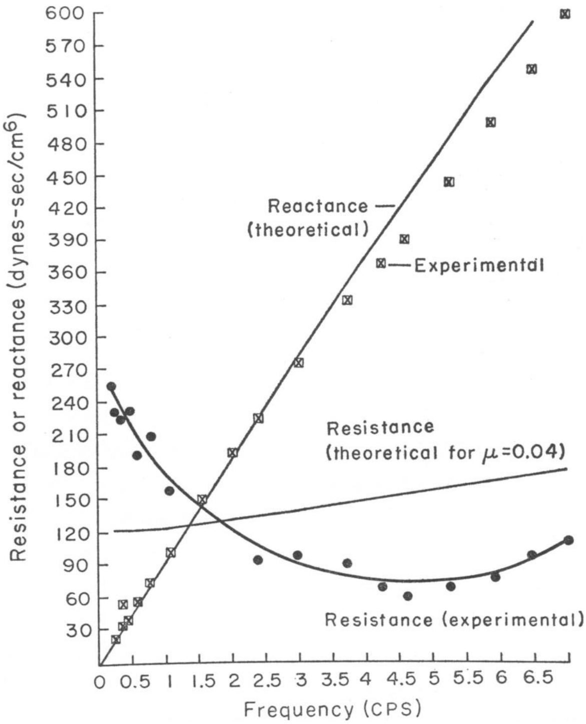

FIGURE 6 Comparison of experimental fluid mechanical resistance and reactance of blood plotted as functions of frequency, with theoretical resistance and reactance of a homogeneous fluid of comparable viscosity. Blood hematocrit, 47%. For explanation see text.

DISCUSSION

Womersley's theory has been criticized (4) on the grounds that it neglects the nonlinear convective terms of the Navier-Stokes equation. However, considerable experimental evidence has now accumulated which indicates that the error so introduced is negligible under most conditions. Thurston (15) found excellent agreement between theory and experiment in tubes of radius 0.0796–1.1340 cm, using water, 10 centistoke, and 100 centistoke fluids. More recently, Linford and Ryan (10) confirmed the theory in a 21 ft pipe 1 inch in diameter, and Shizgal, Goldsmith, and Mason (13) found agreement between observed and theoretical displacement profiles. Our studies provide additional confirmation.

The agreement of our observations using aqueous glycerol with theory has a bearing on another question: could our results for blood be in part due to inlet conditions of the flow tube? Our two pressure taps were each placed at a distance of 30 cm from the respective ends of the flow tube to reduce as much as possible such entrance effects. Since Womersley's theory assumes a tube of infinite length, the agreement of our results for aqueous glycerol with Womersley's theory indicates that entrance effects were indeed negligible in our experiments.

Another question to be considered is: could our results be due in part to reflections of the pressure pulse? In a rigid tube such as ours, where phase velocity and wavelength are large, one would not expect such reflections to be large. Again, the agreement of our observations using aqueous glycerol with Womersley's theory for a tube of infinite length indicates that the contributions of reflected waves were negligible under these conditions.

The use of Womersley's theory to calculate apparent viscosity of blood in oscillatory flow is consistent with the use of Poiseuille's Law under steady flow conditions, since Womersley's equation [equation (3)] reduces to Poiseuille's Law at zero frequency (where $M'_{10}/\alpha^2 = 0.125$). Such a usage is, of course, open to the same objections as the use of Poiseuille's Law for this purpose. Haynes's "differential viscosity" or "generalized viscosity" (6) would be more suitable parameters. However, the three parameters are simply related, as Haynes has shown, and exhibit similar characteristics, at least under steady flow conditions.

Womersley's theory of oscillatory flow in rigid tubes demonstrated that the hydraulic impedance of a Newtonian fluid differs significantly from what one would expect were Poiseuille's Law obeyed instantaneously and continuously throughout the flow cycle, i.e., were the velocity distribution curve always parabolic. Most of the difference is associated with the resistive component of the hydraulic impedance. The resistance in Womersley's theory increases with increasing frequency, whereas the inertance[2] shows little frequency dependence (actually decreasing slightly with increasing frequency). For Poiseuille flow, i.e. flow with parabolic velocity distribution, both resistance and inertance would be independent of frequency.

The studies here reported demonstrate that the hydraulic impedance of blood differs significantly from either Womersley or Poiseuille flow. Again, most of the difference is associated with the resistive component. As shown in Fig. 6, the resistance of blood actually decreases with increasing frequency, at constant stroke volume.

Application of these findings to the circulation must be made with caution. It would appear, however, that the fluctuating viscosity of blood is of negligible sig-

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[2] Inertance, in a hydraulic system, is the opposition of the fluid mass to a change in volume flow rate. Here it is found by dividing the inertial reactance by the angular frequency.

nificance in the large arteries, where the inertance of blood is relatively large compared to the resistance. In this, our findings are consistent with the conclusions of Taylor's theoretical study (14).

The reason for this is most readily seen by referring to equations (8) and (9). The ratio of resistance to inertial reactance is

$$R/X = \tan \epsilon'_{10} .$$

In Womersley's theory, ϵ'_{10} decreases from 90° at $\alpha = 0$ to 0° at $\alpha = \infty$. But $\alpha^2 = r^2\rho\omega/\mu$. At large values of r, α is relatively large, and hence the ratio R/X is relatively small. McDonald (11, p. 90) estimates α (for the fundamental frequency, the heart rate) to be 13.5–16.7 at the root of the aorta in man. Taking 15 as representative, from Womersley's theory $\epsilon'_{10} = 0.097$ and $\tan \epsilon'_{10} = R/X = 0.10$ or 10%. For higher harmonics, R/X would be even less.

The fluctuating viscosity of blood may be more significant in the intermediate and small arteries, where α is smaller and R/X is greater.

Finally, it is of interest to note that the hydraulic impedance of blood appears to be least in the frequency range roughly corresponding to the normal range of heart rate (Fig. 4). While this relationship needs to be further explored, it suggests that the dynamic properties of blood are remarkably well matched to the characteristics of the pumping heart.

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